New Solution Method for Steady-State Canopy Structural Loads

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A new computer code has been written to perform structural analysis of canopies. Although an existing code, CANO, has been available, the new code has better convergence reliability, is more understandably written, and is easier to use. The equations have been reformulated for the new solution method. The new code assumes a symmetric canopy, a steady-state condition, and no strength in the vertical direction. It computes the inflated shape, loads in the horizontal members, radial members, vent lines, suspension lines, and total drag. Constructed geometry, material properties, dynamic pressure, and pressure distribution are required as input.

Nomenclature

l = distance from vent center along radial member
l_h = distance from radial member to center of gore in horizontal direction
l_o = distance from vent center along unstretched radial member
l_{ho} = distance from radial member to center of gore in horizontal direction for unstretched horizontal member
l_{rf} = loaded length of reefing line
l_{sp} = loaded length of suspension line
q = dynamic pressure
r_ψ = gore bulge radius in plane of horizontal member
w_h = constructed horizontal width per radial length
x = axial position of radial member forward from vent
y = radius of radial member from canopy centerline
y_{attach} = radius of attachment point from parachute centerline
y_{skirt} = radius of radial member at skirt from canopy centerline
$\bar{\bar{A}}$ = direction cosine matrix
$C_{\Delta P}$ = pressure differential coefficient
dF_{X_1} = differential force parallel to radial member
dF_{Y_1} = differential force perpendicular to radial member
F_{Yrf} = radial outward force due to reefing line
$dH_{Z_2''}$ = differential force in horizontal member
N = number of gores in canopy
ΔP = pressure differential across canopy
T = tension in radial member
T_{rf} = tension in reefing line
T_{sp} = tension in suspension line
V = velocity
X,Y,Z = axis system at vent center with X along canopy centerline and Y in the plane of the radial member
X_1,Y_1,Z_1 = axis system on radial member with X_1 parallel to the radial member and Y_1 in the plane of the radial member and perpendicular to the radial member
X_1',Y_1',Z_1' = like X_1,Y_1,Z_1 axis system but for the adjacent radial member
X_1'',Y_1'',Z_1'' = axis system whose $X_1''Y_1''$ plane is the plane of symmetry for the gore, and whose $Y_1''Z_1''$ plane contains a horizontal member
X_2'',Y_2'',Z_2'' = axis system whose $Y_2''Z_2''$ plane contains a horizontal member and whose Z_2'' axis is parallel to the horizontal member where it intersects the radial member.
ϵ = strain in radial member
ϵ_h = strain in horizontal member
θ = tangent angle of radial measured from canopy centerline
θ'' = angle of horizontal member tangent plane measured from canopy centerline
θ_{sp} = angle of suspension line measured from canopy centerline
ρ = air density
ψ = gore bulge half-angle
$(\hat{\ })$ = unit vector
$()',()'',()_1,()_2$ = indicates the axis system in which the quantity is measured

Introduction

THE design of a canopy requires knowledge of the loads in the structure to choose materials of the proper strength for its construction. An experienced designer may have the knowledge to choose proper strength materials for a wide variety of applications; however, for other applications, or for fine tuning a design (such as minimizing total canopy weight), an analysis of the canopy loads can be quite valuable. A new computer code, CALA (Canopy Loads Analysis), has been written to do the loads analysis for a wide range of canopy designs. To be of general use, the code must be reliable, reasonably easy to use, and written such that it can be understood by others, both for verification of what equations are solved and for modifying the code for new applications. Previously, a code named CANO[1-4] has been available for canopy loads analysis, but it would not always converge to a correct solution for designs of interest. Furthermore, the input was quite tedious and the internal workings of the code were difficult to understand. A simpler formulation of the problem has been derived and incorporated into the CALA code.

This paper covers four areas pertaining to the CALA code. First, the equations and a description of their derivation will

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be presented. Next, the assumptions used with their strong points and limitations will be discussed. Following will be some comparisons of CALA results to some real data and to some CANO results. Finally, some of the attributes of the CALA code will be included.

Equations for CALA

It is frequently a straightforward task to solve coupled nonlinear ordinary differential equations with modern numerical methods[5] and computers. The equations which are solved in CALA are a set of coupled nonlinear ordinary differential equations. The basic equations that describe a canopy shape will be presented first. It is assumed that all gores are symmetric, and the canopy is in a steady state. Figure 1 shows the geometry of a radial member of a canopy. The X axis is the centerline of the canopy, and the y coordinate is the radius of a point on the radial member. The radial member is assumed to be a line of zero thickness. The equations for change in position are

$$\frac{dy}{dl} = \sin\theta \tag{1}$$

$$\frac{dx}{dl} = \cos\theta \tag{2}$$

where dl is measured along the radial. The change in the tangent angle, θ, is given by

$$\frac{d\theta}{dl} = -\frac{(dF_{Y_1}/dl)}{T} \tag{3}$$

where dF_{Y_1}/dl is the force applied perpendicular to the radial per unit length, and T is the tension in the radial. The change in tension is given by

$$\frac{dT}{dl} = -\frac{dF_{X_1}}{dl} \tag{4}$$

where dF_{X_1}/dl is the force applied parallel to the radial per unit length. These four equations (1–4) are the basic equations that define the shape of a canopy. The perpendicular and parallel forces applied to the radial are assumed to come from the gore structure between radial members and will be detailed later. These forces are defined at each point on the radial and are functions of the position of the radial (x,y), tangent angle (θ), tension (T), and location along the radial (l).

An overly simplified case will be used to illustrate how the forces could be obtained. For now, neglect the gore structure and assume that the pressure differential across the canopy mystically gets applied to the radial member. Further assume that the pressure differential force is perpendicular to the radial, since forces resulting from pressures are normal to the surface upon which they act. Thus,

$$dF_{Y_1} = \frac{2\pi y \Delta P dl}{N}$$

where N is the total number of gores (= number of radials) and the pressure differential, ΔP, is given by

$$\Delta P = qC_{\Delta P} \tag{5}$$

where q is the given dynamic pressure $(\rho V^2/2)$ and $C_{\Delta P}$ is a pressure differential coefficient and is given as a function of l. (It can be argued that $2\pi y$ is not the correct term for a finite number of gores, but this is just an illustrative example.) The force parallel to the radial is zero, since it was assumed that the pressure force was perpendicular and all other forces are ignored. This implies that the tension is constant in the radial for this overly simplified case.

The solution of the four basic differential equations will be an initial value problem if the integration starts at the vent and proceeds to the skirt. At the center of the vent, $x=y=0$ and $\theta=\pi/2$ rad. The tension is unknown, so a guess of the radial tension at the vent center can be made and the integration can proceed along the radial to the skirt. (The vent lines are considered to be part of the radials.) From the resulting solution at the skirt (x,y,θ), the attachment point of the suspension lines to the forebody can be projected (assuming no reefing line for now). If the predicted attachment point radius is too large, the tension can be decreased [see Eq. (3)] to give more curvature to the radial and thus lower the attachment point radius. This is accomplished by decreasing the guess for the radial tension at the vent center. Similarly, if the attachment point radius is too small (it could be projected as negative without affecting the canopy solution), the tension can be increased. Thus, a simple iterative process is used to obtain the tension to match the desired attachment point radius. Figure 2 shows typical radial shapes for the correct tension, and for a tension-too-high case and a tension-too-low case.

The iteration process is started with an initial guess for the vent line tension. The tension is then typically doubled or halved until one guess gives a too-high value and another gives a too-low value. At this point, the high value or the low value may be so ridiculous that the integration could not be performed from the vent all the way to the skirt. If this is the case, then tension guesses are made by splitting the high and low values until integration can go all the way to the skirt, with both the new high and new low guesses. The next step is to do a linear interpolation to predict the vent line tension. Thereafter, a quadratic interpolation is used to predict the vent line tension. If the quadratic interpolation does not predict a value between the latest high and low values, then the interpolation reverts to the linear interpolation. Table 1 shows a typical series of guesses for the vent line tension. Even though the vent line tension is more dependent on differential pressure loading and constructed materials and geometry than on attachment radius, the iteration procedure is precise enough to match a specified attachment radius quite accurately.

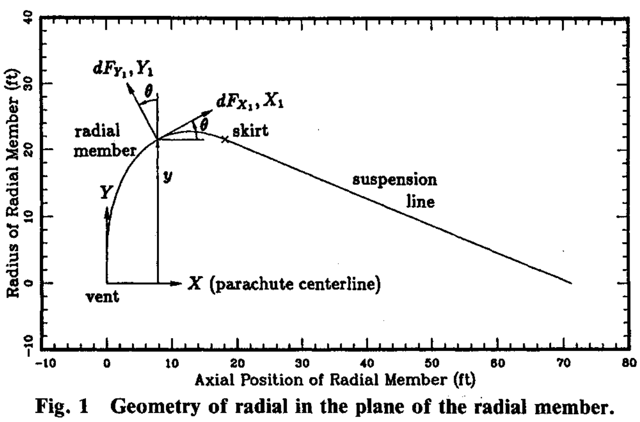

Fig. 1 Geometry of radial in the plane of the radial member.

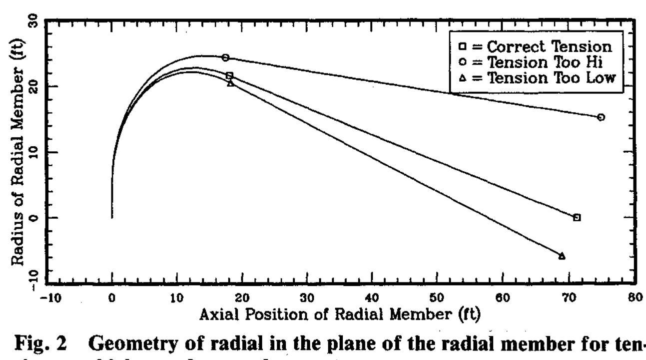

Fig. 2 Geometry of radial in the plane of the radial member for tension too high, too low, and correct.

With the basic idea of the solution method described, a gore structure will be incorporated that includes strength in the horizontal direction (along the gore surface and perpendicular to the gore centerline) but ignores strength in the vertical direction (along the gore surface and parallel to the gore centerline). This involves two parts. One is to determine the forces in the gore structure in an axis system convenient for that purpose. The other is to determine the transformations to relate the forces in the gore structure axis system to an axis system convenient to applying these forces to the radial. The first step then is to define the axis systems. Figure 3 shows an exaggerated view of part of a single gore in a canopy. The planes, which contain two adjacent radial members, form a wedge shape with the centerline of the canopy at the intersection of the planes. The gore to be analyzed lies in this wedge, and the gore is shown from the vent to the current point of interest. Figure 4 shows the same radials and gore as Fig. 3, but the axis systems have been added. The XY axes shown in Fig. 1 are the X and Y axes of the XYZ axis system shown in Fig. 4, and the X_1Y_1 axes shown in Fig. 1 are the X_1 and Y_1 axes of the $X_1Y_1Z_1$ axis system shown in Fig. 4. The plane containing the radial member is the XY plane. The $X_1Y_1Z_1$ axis system has its X_1Y_1 plane in the plane of the radial member, and its orientation is changed from the XYZ axes by a rotation of θ about the positive Z axis, so that X_1 is parallel to the radial, and Y_1 is perpendicular to the radial. The angle θ is the same angle θ shown in Fig. 1. Thus, forces from the canopy structure will be transformed into their components in the $X_1Y_1Z_1$ axis system to be applied to the radial member. The $X_1'Y_1'Z_1'$ axis system is similar to the $X_1Y_1Z_1$ axis system, except that it is for the adjacent radial member on the other side of the gore. It is used later to define the angle θ''.

An axis system must be defined to describe the loads in the gore structure. Since there is no strength in the vertical direction, only a horizontal member of infinitesimal width is considered. First consider an intermediate axis sytem $X_1''Y_1''Z_1''$ which is shown in Fig. 4. Its Z_1'' axis is parallel to a straight line, from the point of intersection of the infinitesimal-width horizontal with the radial, to the point of intersection with the adjacent radial. Its $X_1''Y_1''$ plane is the midgore plane, which is a plane that has been rotated half way (π/N radians) to the plane of the adjacent radial member. This makes the $X_1''Y_1''$ plane a plane of symmetry for the gore. Since there is no strength in the vertical direction, and only a pressure force is included on the canopy, it can be assumed that the infinitesimal-width horizontal member lies in a plane. The $X_1''Y_1''Z_1''$ axis system has been rotated an angle of θ'' about an axis coincident with the positive Z_1'' axis, so that the infinitesimal-width horizontal member is in the $Y_1''Z_1''$ plane. Figure 5 shows a gore in the $Y_1''Z_1''$ plane. The $X_2''Y_2''Z_2''$ axis system is attached to the end of the horizontal member. Its orientation differs from the $X_1''Y_1''Z_1''$ axis system by a rotation of an angle ψ about the X_1'' axis. The infinitesimal-width horizontal member lies in the $Y_2''Z_2''$ plane, and the force applied to the radial member is in the (negative) Z_2'' direction.

Table 1 Iteration process to converge to a zero attachment radius for a 64-ft-diam parachute at a dynamic pressure of 10 psf

Iteration no.	Vent line tension, lb	Attachment radius, in.	Mode to arrive at this guess
0	125.664	−575.140	Initial guess
1	251.327	183.137	Doubled
2	220.977	−68.995	Linear interpolation
3	230.415	18.857	Quadratic interpolation
4	228.286	0.192	Quadratic interpolation
5	228.265	0.003	Quadratic interpolation

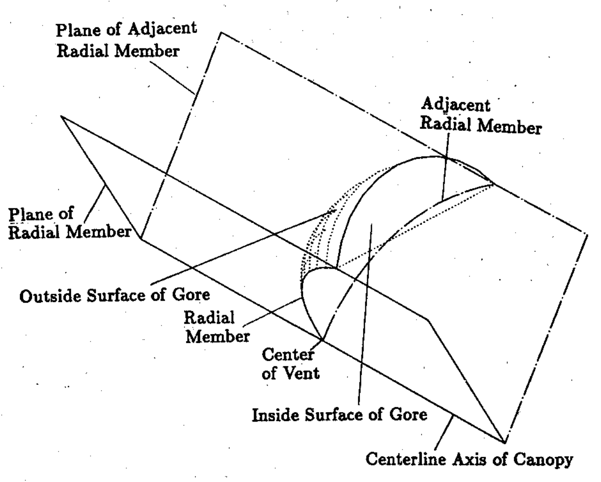

Fig. 3 Exaggerated view of a gore being analyzed shown from vent to the point of interest.

The axes transformations can be derived and written in the form of

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \begin{bmatrix} \bar{\bar{A}}_1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\begin{bmatrix} X_2'' \\ Y_2'' \\ Z_2'' \end{bmatrix} = \begin{bmatrix} \bar{\bar{A}}_2'' \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

where $\bar{\bar{A}}_1$ and $\bar{\bar{A}}_2''$ are 3×3 direction cosine matrices, and then combined to give

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \begin{bmatrix} \bar{\bar{A}}_1 \end{bmatrix} \begin{bmatrix} \bar{\bar{A}}_2'' \end{bmatrix}^T \begin{bmatrix} X_2'' \\ Y_2'' \\ Z_2'' \end{bmatrix}$$

Since the only force in the $X_2''Y_2''Z_2''$ axis system that acts on the radial is in the Z_2'' direction, only the Z_2'' component of force on the radial needs to be included. Thus,

$$\frac{\mathrm{d}F_{X_1}}{\mathrm{d}l} = \left(\sin\psi\ \sin\theta''\ \cos\theta - \sin\psi\ \cos\theta''\ \cos\frac{\pi}{N}\sin\theta + \cos\psi\ \sin\frac{\pi}{N}\sin\theta\right)\left(\frac{\mathrm{d}F_{Z_2''}}{\mathrm{d}l}\right) \tag{6}$$

$$\frac{\mathrm{d}F_{Y_1}}{\mathrm{d}l} = \left(-\sin\psi\ \sin\theta''\sin\theta - \sin\psi\ \cos\theta''\ \cos\frac{\pi}{N}\cos\theta + \cos\psi\ \sin\frac{\pi}{N}\cos\theta\right)\left(\frac{\mathrm{d}F_{Z_2''}}{\mathrm{d}l}\right) \tag{7}$$

$$\frac{\mathrm{d}F_{Z_1}}{\mathrm{d}l} = \left(\sin\psi\cos\theta''\ \sin\frac{\pi}{N} + \cos\psi\ \cos\frac{\pi}{N}\right)\left(\frac{\mathrm{d}F_{Z_2''}}{\mathrm{d}l}\right) \tag{8}$$

The infinitesimal-width horizontal member is assumed to be in the plane normal to a plane that is tangent to the radials on both sides of the gore, where this tangent plane is formed by the X_1 and X_1' axes. This is an appropriate choice, since a pressure differential across the canopy is the only applied force. There are no forces included in this model from vertical strength of the canopy nor from tangential aerodynamic forces that would move the horizontal member from that

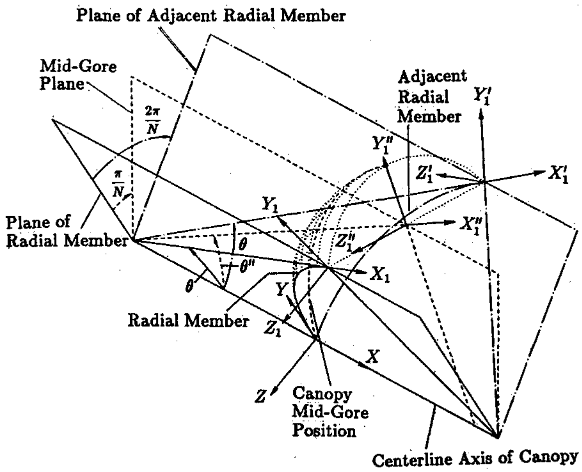

Fig. 4 View of gore being analyzed showing the axis systems.

perpendicular plane. Mathematically, the unit vector $\hat{Y}_1''$ is given by

$$\hat{Y}_1''=\frac{\hat{X}_1\times\hat{X}_1'}{|\hat{X}_1\times\hat{X}_1'|}$$

Equating terms from the above equation with terms in the direction cosine matrix $\bar{\bar{A}}_1''$ gives

$$\sin\theta''=\frac{\sin\theta\cos(\pi/N)}{\sqrt{1-\sin^2\theta\sin^2(\pi/N)}} \tag{9}$$

$$\cos\theta''=\frac{\cos\theta}{\sqrt{1-\sin^2\theta\sin^2(\pi/N)}} \tag{10}$$

Note that θ'' approaches θ as the number of gores becomes large.

Now consider the forces from two gores onto a single radial. First define $\mathrm{d}H_{Z_2''}$ to be the magnitude of the force in the infinitesimal-width horizontal member. For the gore that has been examined, the force acting on the radial is

$$\frac{\mathrm{d}F_{Z_2''}}{\mathrm{d}l}=-\frac{\mathrm{d}H_{Z_2''}}{\mathrm{d}l}$$

To include the gore on the opposite side of the radial, the angle π/N would be replaced by $-\pi/N$, ψ would be replaced by $-\psi$, and $\mathrm{d}F_{Z_2''}/\mathrm{d}l$ would be $+\mathrm{d}H_{Z_2''}/\mathrm{d}l$. Combining the forces for the gores on both sides of the radial in Eqs. (6-8), with the substitution of Eqs. (9) and (10), gives

$$\frac{\mathrm{d}F_{X_1}}{\mathrm{d}l}=-\cos\psi\,\sin\frac{\pi}{N}\sin\theta\left(2\frac{\mathrm{d}H_{Z_2''}}{\mathrm{d}l}\right) \tag{11}$$

$$\frac{\mathrm{d}F_{Y_1}}{\mathrm{d}l}=\left(\frac{\sin\psi\cos(\pi/N)}{\sqrt{1-\sin^2\theta\sin^2(\pi/N)}}-\cos\psi\,\sin\frac{\pi}{N}\cos\theta\right)\left(2\frac{\mathrm{d}H_{Z_2''}}{\mathrm{d}l}\right) \tag{12}$$

$$\frac{\mathrm{d}F_{Z_1}}{\mathrm{d}l}=0$$

Next, equations for loading the horizontal members will be presented. The infinitesimal-width horizontal member is assumed to be an arc from a cylinder of width $\mathrm{d}x_1''$, and a section is shown in Fig. 6. As is seen in Fig. 6, ψ is the gore bulge half-angle and r_ψ is the gore bulge radius. The straight-line distance from radial to radial is given by $2r_\psi\sin\psi$ or alternately by $2y\sin(\pi/N)$ where y is the radius from the canopy centerline to the point of interest on the radial. Then, from Fig. 6,

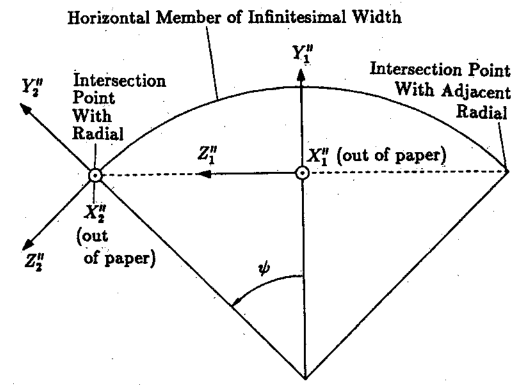

Fig. 5 View of infinitesimal-width horizontal member in its plane.

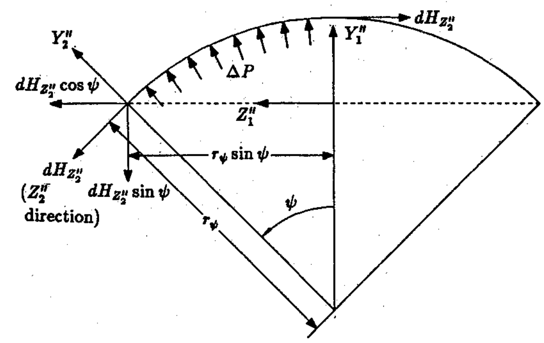

Fig. 6 Loading on horizontal member.

$$\mathrm{d}H_{Z_2''}\sin\psi=\Delta P\left(y\sin\frac{\pi}{N}\mathrm{d}x_1''\right) \tag{13}$$

and the area that is multiplied by the pressure differential is independent of horizontal member shape. The angle ψ can be determined by using the arc length of the horizontal member. Letting l_h be the arc distance from a radial to the midgore position,

$$r_\psi=\frac{l_h}{\psi}$$

and since

$$\frac{l_h}{\psi}\sin\psi=y\sin\frac{\pi}{N}$$

then

$$\frac{\sin\psi}{\psi}=\frac{y\sin\pi/N}{l_h} \tag{14}$$

which must be solved iteratively, where l_h is a obtained from canopy geometry (which is a function of l) and horizontal loading for an elastic material. Since $\hat{X}_1$ (the direction of $\mathrm{d}l$) and $\hat{X}_1''$ are slightly different directions, the direction cosine relating these two directions must be included in determining $\mathrm{d}H_{Z_2''}/\mathrm{d}l$. Combining Eq. (13) and the direction cosine term [with Eq. (9) and (10)] gives

$$\frac{dH_{Z_2''}}{\mathrm{d}l}=\frac{\Delta Py\sin(\pi/N)}{\sin\psi}\sqrt{1-\sin^2\theta\sin^2\frac{\pi}{N}} \tag{15}$$

At this point, a complete set of equations (1-5, 11, 12, 14, and 15) has been stated to solve for canopy shape and loads in the radial and horizontal members if there is no stretch in the canopy. For stretch in the canopy members, it is more convenient to work in terms of the unstretched radial length as the independent variable. Let l_o represent a distance along the unstretched radial, and l be the stretched distance. The relationship between l and l_o is given by

$$\frac{\mathrm{d}l}{\mathrm{d}l_o} = 1 + \epsilon \tag{16}$$

where ϵ is the strain in the radial member and

$$\epsilon = f(T, l_o) \tag{17}$$

where the dependence on l_o is to give the strength characteristics of the radial as a function of position. Equation (16) can be multiplied by each of Eq. (1-4) to change the basic equations to use unstretched radial length as the independent variable. Including the effect of stretch on the horizontal members requires some care. First, the strength of a horizontal member per unit length along the stretched radial member decreases as the radial stretch increases. Second, the width of a horizontal member may or may not increase with radial member stretch. For a solid canopy, it is assumed that the width of the horizontal member increases with radial stretch. For a ribbon canopy, it is assumed that the width of the horizontal member stays constant when the radial stretched. Thus, Eq. (15) is rewritten as

$$\frac{\mathrm{d}H_{Z_2''}}{\mathrm{d}l} = \frac{\Delta P y \sin(\pi/N)}{\sin\psi} \min\left(\sqrt{1 - \sin^2\theta \sin^2\frac{\pi}{N}},\; \frac{\mathrm{d}w_h}{\mathrm{d}l_o}\frac{\mathrm{d}l_o}{\mathrm{d}l}\right) \tag{18}$$

where $\mathrm{d}w_h/\mathrm{d}l_o$ is the constructed horizontal width per unit constructed radial length. (The cosine of the angle between the centerline of the gore and the radial member is w_h when the local area of a single gore is laid flat.)

Equation (14) is solved iteratively. An initial guess for ψ is necessary to start the iteration process. Since the solution of the canopy shape is proceeding from the vent to the skirt, usually the last converged value of ψ is used to start the next iteration. If no value is available, then ψ is set to 1 rad. To control the iteration, minimum and maximum allowable values for ψ are set, which are changed during the iteration process when new minimum and maximum values are determined. Initially $\psi_{\min} = 0$ and $\psi_{\max} = \pi$. The actual iteration process begins by calculating the load in the infinitesimal-width horizontal member using Eq. (15) or (18). Next the strain, ϵ_h, in the horizontal member is determined from

$$\epsilon_h = f\left(\frac{\mathrm{d}H_{Z_2''}}{\mathrm{d}l}, l_o, \frac{\mathrm{d}l}{\mathrm{d}l_o}\right) \tag{19}$$

where the dependence on l_o is to give the strength characteristics of the horizontal as a function of position, and the dependence on $\mathrm{d}l/\mathrm{d}l_o$ is the horizontal strength reduction per radial length with radial stretch. The arc distance for the stretched horizontal is given by

$$l_h = l_{ho}(1 + \epsilon_h) \tag{20}$$

where l_{ho} is the constructed horizontal half-gore-width length. Equation (14) is rewritten to define the error in the iteration as

$$e = \frac{y \sin(\pi/N)}{l_h} - \frac{\sin\psi}{\psi} \tag{21}$$

where e is the error. If the error is not sufficiently close to zero, then a correction to ψ is computed from

$$(\Delta\psi)_{\text{comp}} = -e\Big/\left(\frac{\partial e}{\partial\psi}\right) \tag{22}$$

where

$$\frac{\partial e}{\partial \psi} = \frac{y \sin(\pi/N)}{l_h(1+\epsilon_h)} \frac{\partial \epsilon_h}{\partial\left(\frac{\mathrm{d}H_{Z_2''}}{\mathrm{d}l}\right)} \left(\frac{\mathrm{d}H_{Z_2''}}{\mathrm{d}l}\right) \frac{\cos\psi}{\sin\psi} - \frac{(\psi\cos\psi - \sin\psi)}{\psi^2} \tag{23}$$

$$\frac{\partial \epsilon_h}{\partial\left(\frac{\mathrm{d}H_{Z_2''}}{\mathrm{d}l}\right)} = f\left(\frac{\mathrm{d}H_{Z_2''}}{\mathrm{d}l}, l_o, \frac{\mathrm{d}l}{\mathrm{d}l_o}\right) \tag{24}$$

If $(\Delta\psi)_{\text{comp}} > 0$, indicating that ψ needs to be increased, then $\psi_{\min}$ is set to the current value of ψ, and

$$\Delta\psi = \min[(\Delta\psi)_{\text{comp}}, 0.75(\psi_{\max} - \psi)] \tag{25}$$

If $(\Delta\psi)_{\text{comp}} < 0$, then $\psi_{\max}$ is set to the current value of ψ, and

$$\Delta\psi = \max[(\Delta\psi)_{\text{comp}}, 0.75(\psi_{\min} - \psi)] \tag{26}$$

The final step is

$$\psi_{n+1} = \psi_n + \Delta\psi \tag{27}$$

and the iteration is repeated.

Analysis of Reefed Configuration

Conceptually, a reefed configuation is treated the same as an unreefed configuration. An obvious change is that the pressure distribution specified must be suitable for the reefing ratio being analyzed. Frequently, the length of the uninflated portion of the canopy can be obtained from photos of reefed parachutes, and this information can be incorporated into the specified pressure distribution.

The mathematical procedure for projecting an attachment point radius is altered when a reefing line is included. The stretched reefing line length l_{rf} is calculated from

$$l_{rf} = 2Ny_{\text{skirt}} \sin\frac{\pi}{N} \tag{28}$$

where y_{skirt} is the radius of the radial at the skirt. The tension in the reefing line T_{rf} is a function of l_{rf}. The force in the radial outward direction due to the reefing line F_{Yrf} is given by

$$F_{Yrf} = -2T_{rf} \sin\frac{\pi}{N} \tag{29}$$

The suspension line tension T_{sp} and angle θ_{sp} can be determined from axial (drag) and radial force balances, which are given by

$$T_{sp} \cos\theta_{sp} = T\cos\theta \tag{30}$$

$$T_{sp} \sin\theta_{sp} = T\sin\theta - F_{Yrf} \tag{31}$$

The loaded suspension line length l_{sp} is a function of T_{sp} and the attachment point radius y_{attch} is projected from

$$y_{\text{attch}} = y_{\text{skirt}} + l_{sp} \sin\theta_{sp} \tag{32}$$

The iteration scheme currently used in CALA for a reefed parachute is the same as for an unreefed configuration, except that the high value and low value for the vent tension guesses must both be in a range that gives realistic reefing line strains before the interpolation phase is begun. This is much more restrictive than just being able to integrate from the vent to the skirt, which was the only requirement for an unreefed configuration. This generally means making several extra iterations using the splitting phase for the vent tension guess, but this method has been found to be robust. Some additional logic in the code could give a more efficient iteration scheme.

Consequences of Assumptions in CALA

The CALA code includes the most important aerodynamic loading, which is the pressure differential across the horizontal members. It also calculates loads in the major structural elements, i.e., the radial members, horizontal members, vent lines, suspension lines, and a reefing line if present.

There are cases where the assumptions used in CALA are limiting for the case being analyzed. An obvious limitation of CALA is that it does only a static analysis, and, in many cases, dynamic effects are important. The loading due to inertial effects is totally ignored in CALA, and, frequently, peak loads occur sometime during the inflation process. Several aerodynamic forces are ignored in CALA. These include all forces on suspension lines, vent lines, and radial and vertical members in the gaps beween horizontals. Additionally, skin friction and flag drag are ignored on the horizontal members. The other aerodynamic forces become more important for highly reefed canopies.

The effect of ignoring vertical strength is twofold. The obvious is that no loading information is calculated for vertical members. The other effect is that the gore shape is determined by the vertical members. CALA assumes the pressure acts normal to the plane that is tangent to both radials, where, in fact, the pressure acts normal to the canopy surface. These directions are frequently very close, but an example where a noticeable difference can occur is for a ring sail canopy. The error would affect the total drag calculation. The vertical strength needs to be included for a loads balance that includes all the aerodynamic forces on the canopy.

In the vent region, especially for canopies with a large number of gores, the width of the radial member may be a significant fraction of the gore width. In such cases, the actual gore geometry will be different than the computed gore shape, which assumes a zero-width radial member. The corresponding loads in the horizontal members may then be different. For cases with a very large gore bulge half-angle ($\psi > 90$ deg), CALA makes no attempt to resolve the interference between gores. CALA does not allow double load paths, which means that pocket band or flared skirt loading cannot be analyzed. A vent pull down line cannot be analyzed, but some provisions to add that capability to CALA have been included.

It would be a reasonably straightforward addition to CALA to include aerodynamic forces on the suspension lines, vent lines, and radial and vertical members in the gaps if these forces were specified as normal and tangential force coefficients. Addition of tangential or flag drag forces on the horizontal members would require rethinking the assumed orientation (θ'') of the infinitesimal-width horizontal member between radials.

Comparison of CALA to Other Sources

The CALA code has been compared to data from a 3-ft diam ribbon parachute that was tested in the LTV-LSWT wind tunnel in February 1974. The particular test is one of several described by Pepper and Reed.[6] The detailed data reduction was done by Heinrich and Uotila,[7] and the comparison is with case 98.1 of their report. That case is a 15% porosity ribbon parachute at a dynamic pressure of 75 psf. The measured drag in the wind tunnel was 303.5 lb, and the drag predicted from the CALA code is 267.0 lb. This represents an error of −12%. Remember that only the drag due to pressure differential on the horizontal members is included by CALA. The computed and measured shapes are compared in Fig. 7. The measured shape would be the view of the midgore shape rather than the radial shape.

A comparison is also made with the CANO code for a 64-ft-diam solid canopy parachute with a total predicted drag force of 10,800 lb. [CANO requires a total drag force as input, whereas CALA calculates a total drag force from an input pressure. The pressure distribution must be modified (q or $C_{\Delta P}$) to match a specific total drag force with the CALA code.] The shape comparison is given in Fig. 8. The loading comparison is given in Fig. 9 for the loads in the radial member, and in Fig. 10 for the loads in the horizontal members (horizontal direction of the gore). Figures 8 and 9 show the codes give nearly identical results for shape and radial load. Figure 10 shows the horizontal load predicted by CANO to be 15% less than that predicted by CALA.

Attributes of CALA Code

The CALA code has some automated geometry generation features that reduce the amount of input necessary to run the code. The specification of a gap and a horizontal member (or a horizontal member alone) can be repeated with gore width being automatically incremented. The gore width can be automatically incremented based on a constructed cone angle that can be changed for different parts of the canopy. For ribbon canopies, the horizontal ribbon lengths can be based on a

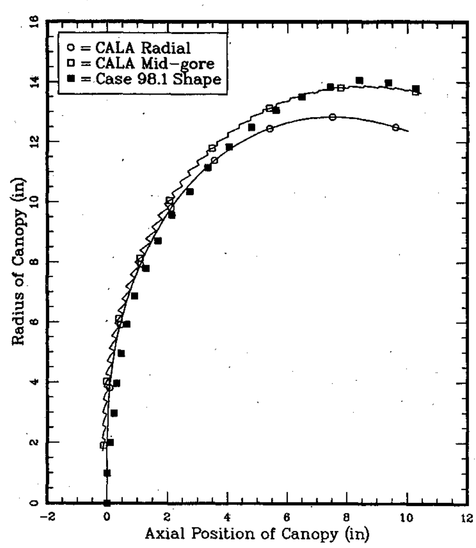

Fig. 7 Comparison of shape predicted by the CALA code with wind-tunnel model case 98.1.

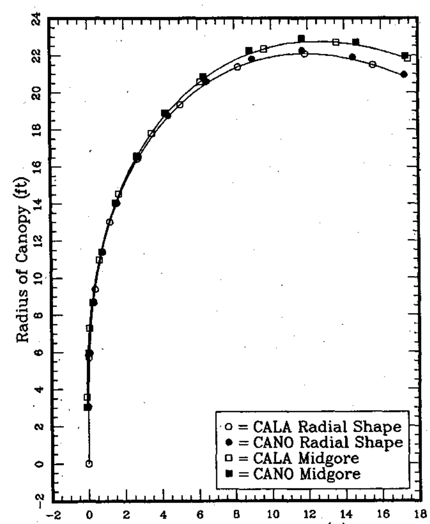

Fig. 8 Comparison of shape predicted by the CALA and CANO codes for a 64-ft-diam solid canopy parachute at a dynamic pressure of 5.8 psf.

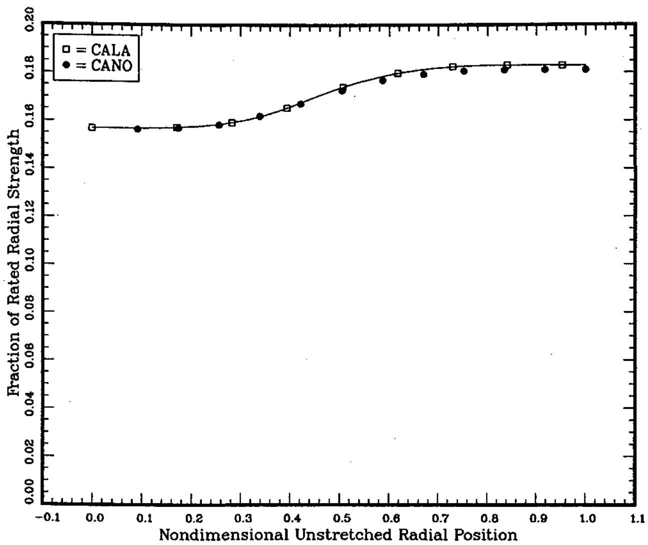

Fig. 9 Comparison of radial loads predicted by the CALA and CANO codes for a 64-ft-diam solid canopy parachute at a dynamic pressure of 5.8 psf.

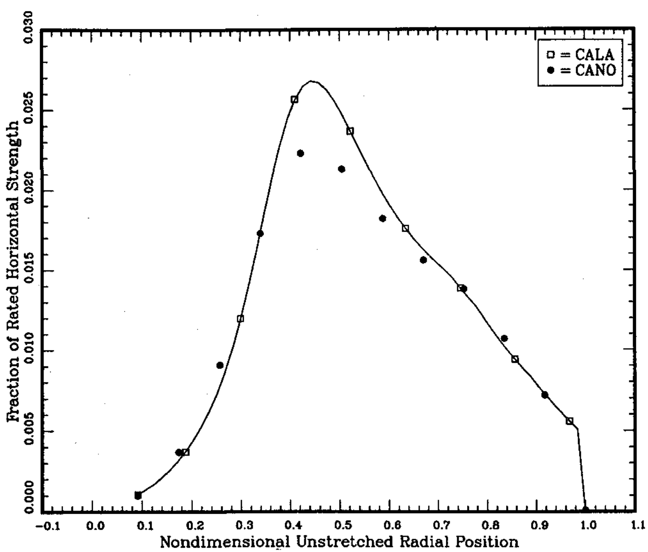

Fig. 10 Comparison of horizontal loads predicted by the CALA and CANO codes for a 64-ft-diam solid canopy parachute at a dynamic pressure of 5.8 psf.

continuous ribbon design (vent-edge dimension equal to skirt-edge dimension) or an individually cut gore design (vent-edge dimension shorter than the skirt-edge dimension). Horizontal element lengths can be automatically determined for a spherical constructed shape (hemisflo canopy) or for a section of a torroidal shape in a part of the canopy. The rated strength of the horizontal members can be specified per segment (convenient for ribbon canopies), or per horizontal member width, or per length measured along the radial member.

Contrasting this part of the input to the input for CANO, CANO either required each horizontal element can be specified individually (and in some cases three elements were input to specify a single physical element), or some specifically modified versions of CANO were available for automatic geometry generation for a single type of canopy, such as a ribbon canopy with a single constructed cone angle and even ribbon spacing. The accuracy of a CANO model was related to the input element size, where in CALA the accuracy is determined by the ordinary differential equation solving routine and accuracy is not decreased by specifying large horizontal segments for sections of solid canopy, although solution printout points are determined by horizontal segments. CALA requires the dynamic pressure as input and then calculates the drag, where CANO required the drag load as input.

The solution method in CALA is inherently consistent at the vent, which is generally the highest loaded area of the canopy. It is very important to have the shape of the canopy correct at the vent, because a small error in position can give a large error in strain and loading on a structural member, especially for a low strain material such as Kevlar. The CANO solution method started at the skirt and then tried to match boundary conditions at the vent, which means a small error in position at the vent could be a large error in structural loading. This has been observed in CANO solutions. Furthermore, CANO varies two parameters at the skirt to try to converge to a solution at the vent, where CALA varies at single parameter (vent line tension) to match an attachment radius. The convergence shown in Table 1 is typical. Also, a small error in attachment radius is insignificant.

Conclusion

The equations to determine the structural loading in a canopy have been presented and, using numerical methods, the loading in a canopy can be obtained. The CALA code is a working code that converges for realistic problems and eliminates errors in the vent region due to convergence problems. CALA has automatic geometry generation capability and is general enough for most symmetric canopies. Including only the horizontal member strength in the canopy and only the pressure differential force on the horizontal member is adequate for many cases. The inclusion of vertical strength in the canopy would be an important major addition to the code. Several minor additions to the code could improve its accuracy and versatility.

Acknowledgment

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